Separation of Proof and Program

The Trellys Project

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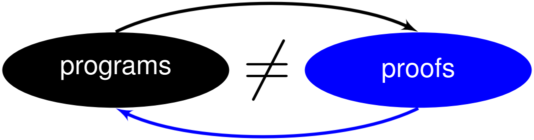
Introduction

- The design of a core language of a new dependently typed programming language Trellys.
  - Separation of Proof and Program (Sep^3).
- The logical fragment.
  - Equality, explicit conversions, a new termination predicate, case splitting on programs, and induction using a primitive ordering.
- The programmatic fragment.
  - General recursion, explicit conversions, and case splitting on programs.

Sep^3

- Sep^3 is a call-by-value language that consists of two fragments, a logical fragment and a programmatic fragment.
  - The language syntactically separates the logical and programmatic fragments.
  - The logical fragment is a predicative higher-order logic.
  - While the programmatic fragment contains general recursion and type:type.

- The two fragments are separate, but they are linked.
  - Proofs can "talk" about programs, but are not allowed to run them.
  - Programs can contain proofs.



The Logical Fragment

- A predicative higher order logic.
  - The logic is weakly constructive. What this means is that there is only one predicate that forces the logic to be non-constructive.
- The logical fragment is compile time only.
  - That is all proofs are erased during compile time.
- The logic has the following as primitives.
  - Disjunction, existential types, absurdity, higher-order predicative quantification, implication, propositional equality, explicit conversions, induction, and a new termination predicate.

Equality and Conversion

- The logic of Sep^3 has a primitive notion of propositional equality.
  - This equality is a typed equality and expresses when two programs are equivalent.
  - Intro. form: $\Gamma \vdash join\ n\ m : t_1 = t_2$.
  - Use: Suppose $|t|$ is a function that erases all the proofs from the program t then we if $|t_1| \leadsto^n t'$ and $|t_2| \leadsto^m t'$ and t_1 and t_2 are typeable then we may conclude that t_1 and t_2 are equivalent with the proof *join*.

- Explicit conversion adds the ability to make use of equalities between programs.
  - Elim. form: $\Gamma \vdash conv\ p\ by\ eqpf\ at\ hole.p : [t_2/hole]P$
  - Use: If we know p is a proof of $[p_1/hole]P$, and we can prove $t_1 = t_2$ then we can replace t_1 with t_2 in $[p_1/hole]P$ and obtain a new proof of $[p_2/hole]P$.

Termination

- The logic contains a new predicate called the termination predicate.
  - The termination predicate internalizes the notion of termination.
  - Predicate form: $t!$.
  - Explanation: For some program t if we can prove t normalizes then we may conclude $t!$.
- We not only need to show that t normalizes, but that the normal form of t can be judged a value.
  - Intro. form: $\Gamma \vdash \textit{valax}\ t : t!$.
  - Use: If we can judge t a value, denoted $\Gamma \vdash \textit{val}\ t$, then we may conclude with the proof $\Gamma \vdash \textit{valax}\ t : t!$ which states that t has a value.

  - Forms that may be judged values:
    - λ-abstractions, *Type*, recursors, data type constructors whose arguments are values, and variables marked as values.

Termination: An Example

Example

Suppose t is a program and v is t's value. Then

$$\dfrac{\dfrac{\begin{array}{l}|t| \downarrow |v| \\ \Gamma \vdash t : t' \\ \Gamma \vdash v : t'\end{array}}{\Gamma \vdash \mathit{join}\ m\ n : t = v} \qquad \dfrac{\Gamma \vdash \mathit{val}\ v}{\Gamma \vdash \mathit{valax}\ v : [v/\mathit{hole}](\mathit{hole}!)}}{\Gamma \vdash \mathit{conv}(\mathit{valax}\ v)\ \mathit{by}\ (\mathit{join}\ m\ n)\ \mathit{at}\ \mathit{hole}.(\mathit{valax}\ \mathit{hole}) : t!}$$

Termination

- How can we use the termination predicate?
  - If p is a proof of $t!$ for some program t then t can be used as a value.
    - Form: $\Gamma \vdash val\ tcast\ t\ by\ p$.
  - *tcase* provides the ability to case split on the termination behavior of programs.
    - DISCLAIMER: $t!$ is not constructive.
    - Form: $\Gamma \vdash tcase\ t\ [u]\ of\ abort\ \rightarrow p_1 \mid ! \rightarrow p_2 : P$.
    - Use: For some program t if p_1 is a proof of some predicate P assuming $t!$ and p_2 is a proof of P assuming t diverges then $tcase\ t\ [u]\ of\ abort\ \rightarrow p_1 \mid ! \rightarrow p_2$ is a proof of P.

Induction

- Sep^3 has a primitive notion of structural course-of-values induction.
  - Form: $\Gamma \vdash \mathit{ind}\ f\ x : t_1, u.p : \forall x : t_1.\forall u : x!.P$.
  - Use: If p is a proof of some predicate P assuming $\forall y : t_2.\forall u : y < x.[y/x]P$ holds then we can prove P for any program x of type t_1.

The Programmatic Fragment

- The programmatic fragment has a collapsed syntax. Terms and types are all generated by the same syntactic category.
- Type:Type
- General recursion.
  - Form: $\Gamma \vdash rec\ f\ x : t_1.t : \Pi x : t_1.t_2$.
- Explicit conversions.
- Data types and case splitting on programs.
  - Intro. Form: $data\ C\ t\ where\ \{C_1 : t_1, \ldots, C_n : t_n\}$.
  - Elm. Form: $case\ t\ [eq\_pf]\ of\ C_1\ t_1 \ldots t_n \mid \cdots \mid C_k\ t'_1 \ldots t'_2 \mid done$.

Concluding remarks

- Future work.
  - Complete the meta-theory.
  - Design and implement the surface language.
- Thank you all for listening.